What is the Relationship between Pressure & Volume Change in a Magma Chamber and Surface Deformation at Active Volcanoes?

What factors control the magnitude of surface deformation?

Using surface deformation data to investigate pressure and volume changes in magma chambers

<u>Core Quantitative Issue</u>
Goodness of Fit

<u>Supporting Quantitative Issues</u>
Parameter Estimation
Dimensional analysis

SSAC – Physical Volcanology Collection
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Preview

This module presents a theoretical model for the relationship between changes in volume and pressure in a magma chamber and measured surface displacements.

Slides 3-6 *give some background on volcano deformation monitoring and modeling approaches.*

Slides 7 and 8 *state the problem. What is the relationship between pressure and/or volume change and surface deformation measured at active volcanoes?*

Slides 9 and 10 *develop a plan for solving the problem. You will rely on data gathered prior to the 2010 eruption at Volcano X and estimate the change in pressure and volume of a magma chamber and the magma chamber depth using the Mogi equations.*

Slides 11-13 *illustrate the solution of the problem, develop a spreadsheet to calculate the models and estimate the goodness-of-fit of data to the model.*

Slide 14 *discusses the point of the module and provides a broader volcanological context.*

Slide 15 *consists of some questions that constitute your homework assignment.*

Slide 16-18 *are endnotes for elaboration and reference.*

Background

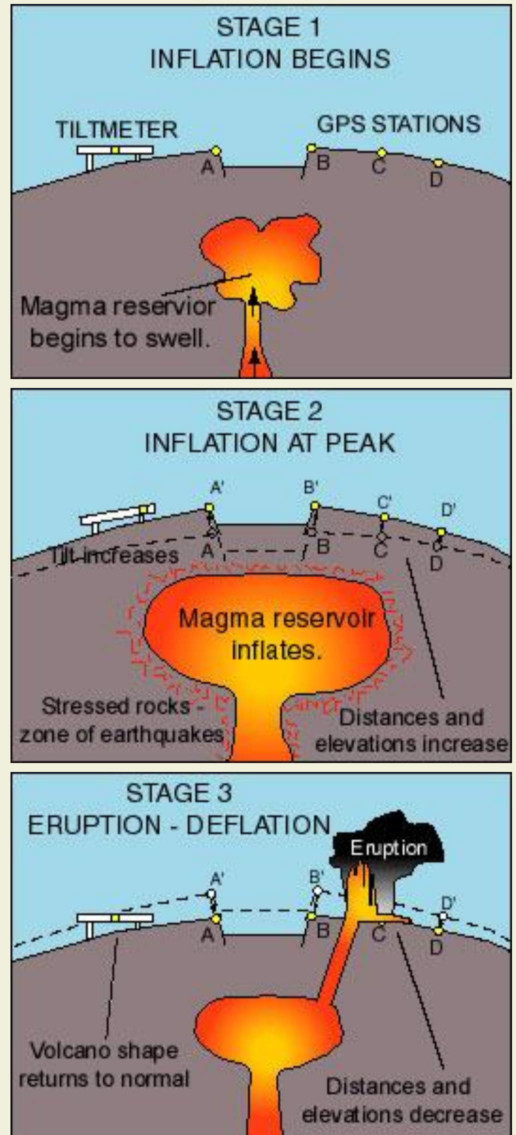
What is surface deformation?

Volcanoes often exhibit geophysical or geochemical signals before, during and after an eruption. These signals allow volcanologists to monitor active volcanoes to gain knowledge about processes in the magma chamber, conduit and edifice and to potentially predict the time and location of a future eruption.

One type of geophysical signal is the deformation or movement of the volcanic edifice and surrounding crust. Changes in the surface of the volcano are related to magma intrusion, dome growth, pressure increase in the magma chamber or flank instability.

For more information about surface deformation:
http://hvo.wr.usgs.gov/howwork/subsidence/main.html

Possible magma intrusion scenario with surface uplift, followed by eruption and subsidence.



Stage 1: Inflation begins as magma moves into the volcano or as pressure increases in the magma chamber.

Stage 2: As the magma chamber inflates, the ground surface above it is displaced.

Stage 3: After an eruption, the magma chamber deflates. Ground surface subsides with potential formation of a crater.

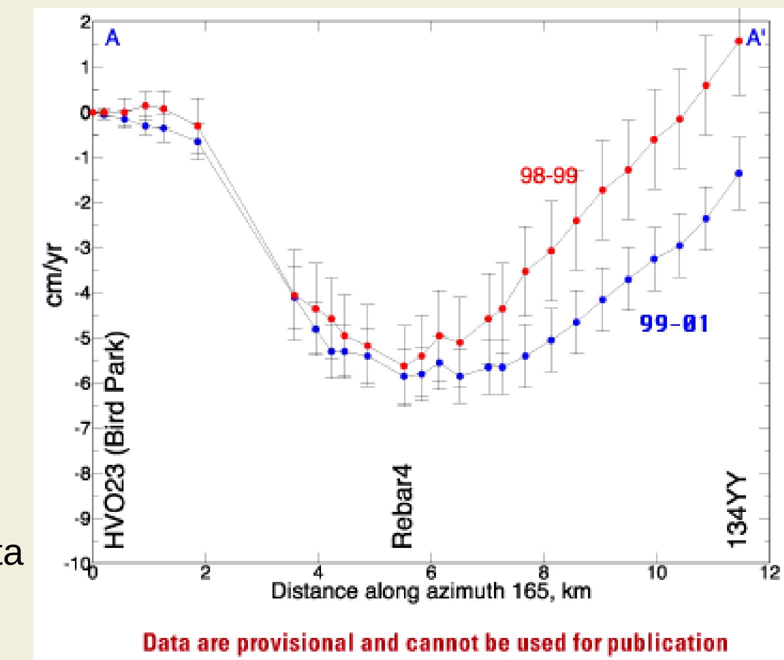
Background

What is the magnitude of surface deformation on volcanoes?

(Above) Mount St. Helens, May 12, 1980. A cryptodome intruded into the north flank of the volcano causing a “bulge” to form. By this date the “bulge” was growing at 1.5 - 2 m per day. Photo USGS.

(Right) Tilt data across Halemaumau crater, Kilauea volcano for the periods 1998-99 & 1999-2001. These data indicate near symmetrical subsidence at rates of cm yr^{-1}. Image USGS.

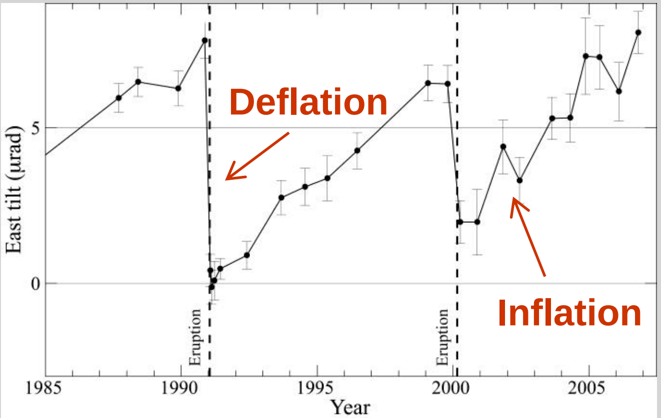
The magnitude of surface deformation (i.e., the amount of vertical (uplift or subsidence) and horizontal displacement) of a volcanic edifice is related to the geometry and depth of the source and the source strength. For example, ***cryptodome intrusion*** (change in *volume* at shallow levels) can cause the flank of a volcano to “bulge” out and become unstable. This can happen at rates of meters per day, as in the case of Mount St. Helens (left), with total displacements of 100’s of meters. Where as, an ***increase or decrease in pressure*** in a magma chamber can cause near symmetrical uplift of the volcano with total magnitudes of meters prior to or after an eruption (below).



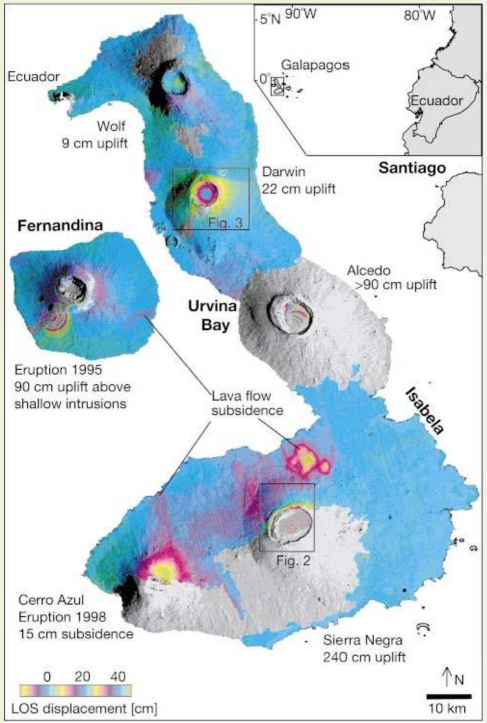
Background

How do we measure volcano deformation?

Investigation of the shape of the Earth is called *geodesy*. Volcanologists employ various terrestrial and satellite based *geodetic* methods to measure changes in the surface of volcanoes. Terrestrial or ground based methods include leveling, tilt and/or electronic distance measurements (EDM). Satellite geodetic methods include the Global Positioning System (GPS) and Interferometric Synthetic Aperture Radar (InSAR).



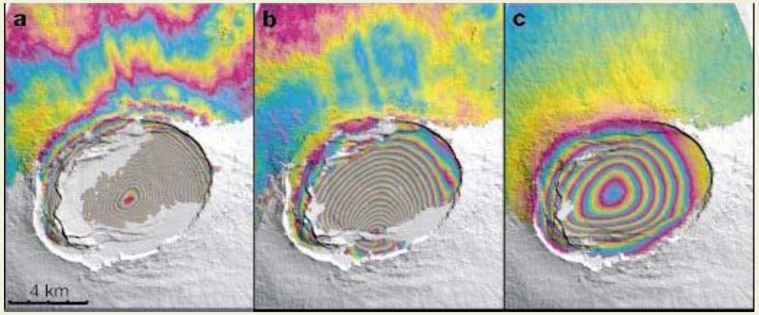
Tilt measured west of Hekla volcano, Iceland indicating pre-eruptive inflation, followed by co-eruptive subsidence for the last two eruptions.
Plot provided by E. Sturkell.



Left: InSAR analysis of Fernandina Island, Galapagos Islands, Ecuador (from Amelung et al. 2000).

Learn more about geodesy:
http://en.wikipedia.org/wiki/Geodesy

For more about monitoring volcano deformation:
http://volcanoes.usgs.gov/About/What/Monitor/Deformation/GrndDefrm.html



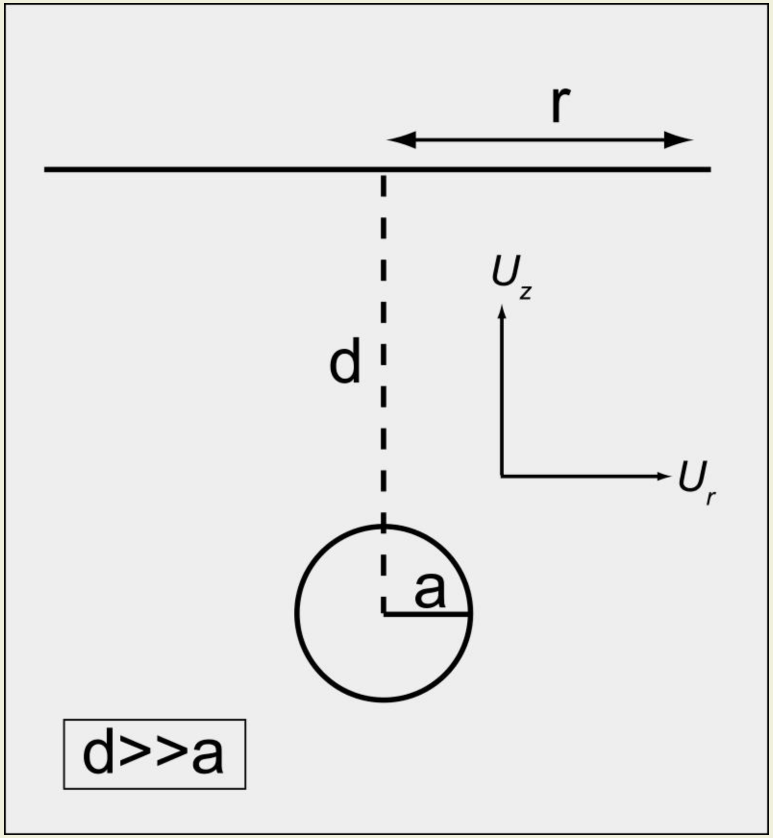
Detailed InSAR images of Sierra Negra volcano, showing uplift between 1992-97 (a), 97-98 (b) and 98-99 (c). Each color fringe represents 2.83 cm change in elevation.

Background

Can we learn something about processes within a volcano from surface deformation data?

In 1958, Kiyoo Mogi from the Earthquake Research Institute, Japan, investigated the patterns of surface deformation associated with two historically active volcanoes, Sakurajima volcano, Japan and Kilauea volcano, Hawaii (Mogi, 1958).

Using the theory of elasticity, Mogi suggested that the surface deformation measured before and after eruptive activity could be fit by a spherical pressure source buried in an elastic medium. In this model, changes in pressure ($\Delta \boldsymbol{P}$) or volume ($\Delta \boldsymbol{V}$) within a magma chamber of radius (***a***) and at depth (***d***) could cause vertical ($\boldsymbol{U_z}$) and horizontal ($\boldsymbol{U_r}$) displacements at the surface. The magnitude of the displacements changes with radial distance (***r***) away from the center. This model is valid when ***a* << *d*** and has been called the Mogi point source model.



Learn More about the Mogi Model:
http://www.geophys.uni-stuttgart.de/oldwww/ew/volcano/santorin.html

Problem

What is the relationship between pressure and/or volume change in a magma chamber and surface deformation?

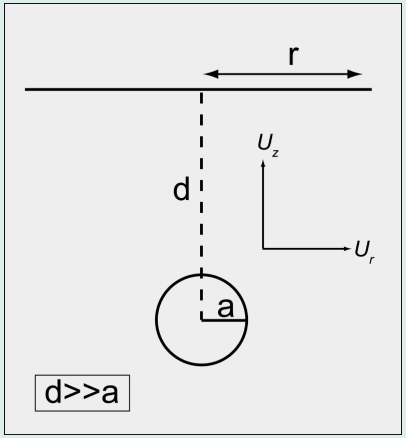
The relationship between surface displacements and pressure change for a spherical source in an elastic half-space from Mogi (1958):

Vertical Displacement (uplift or subsidence):

$$U_z = \frac{3a^3 \Delta P d}{4G(d^2 + r^2)^{1.5}}$$

Horizontal Displacement:

$$U_r = \frac{3a^3 \Delta P r}{4G(d^2 + r^2)^{1.5}}$$



where (*a*) is the magma chamber radius (500-1000 m), (Δ*P*) is pressure change in the chamber (10-40 MPa), (*G*) is the elastic shear modulus or rigidity (30 GPa or 3x10^{10} Pa), (*d*) is the depth to the center of the magma chamber (3-10 km), and (*r*) is the radial distance from the point source (0-50 km).

Make sure you can see that the units of U_z and U_r are meters, given the Mogi equations.

Problem (continued)

What is the relationship between pressure and volume change in a magma chamber and surface deformation?

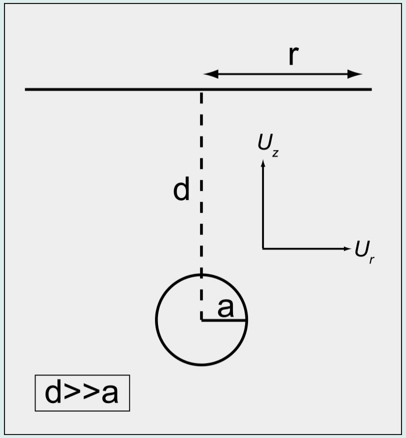
The relationship between surface displacements and volume change from Mogi (1958):

Vertical Displacement (uplift or subsidence):

$$U_z = \frac{3\Delta V d}{4\pi(d^2 + r^2)^{1.5}}$$

Horizontal Displacement:

$$U_r = \frac{3\Delta V r}{4\pi(d^2 + r^2)^{1.5}}$$



Parameters are the same as in the equations for pressure change, but now we look at volume change, ΔV (m^3). Volumes of historical eruptions range from 1 m^3 to 15 km^3.

Learn more about where this equation comes from

Designing a Plan, Part 1

Given surface displacement data for uplift across Volcano X, use the Mogi point source equation to solve for the change in pressure and depth of the magma chamber.

You will need to:

- **Calculate the surface uplift versus radial distance for variable pressure and volume sources.**
- **Calculate the "goodness of fit" of your model compared to the data.**

You will consider the goodness-of-fit of your model using the Pearson's χ^2 test.

***Notes*:**

Data for the uplift of Volcano X were collected prior to the 2010 eruption.

You will calculate the "best-fit" model for pressure change and depth of the magma chamber.

You will use Pearson's Chi-Square test to estimate the goodness of fit of your model to data:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

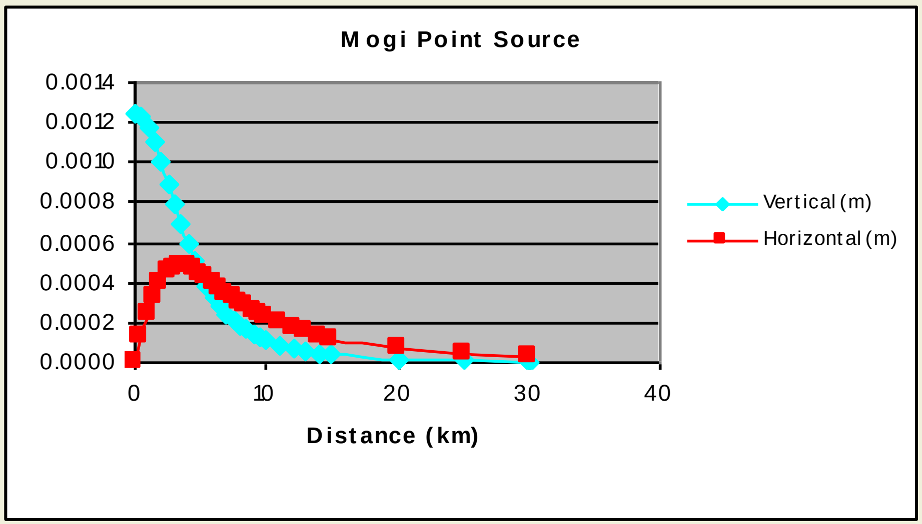
where O_i is the observation and E_i is the expected value or model result.

Learn more about Pearson's chi-square test:
http://en.wikipedia.org/wiki/Pearson%27s_chi-square_test

Designing a Plan, Part 2

Mogi (1958) utilized elasticity theory to demonstrate that uplift and subsidence measured at Sakurajima and Kilauea volcanoes were related to changes in pressure in a magma chamber.

You will utilize his equations to investigate the relative importance between the depth and strength (pressure or volume change) of the source.



Cell with a number. Change one of these numbers and other numbers will change.

Cell with an equation. It's up to you to determine the equation that produces the number that appears in the cell.

Create an Excel spreadsheet using given values (yellow cells). Use formulas to calculate the correct vertical and horizontal displacements.

	B	C	D	E
2	***Calculate a Mogi Point Source for Surface Deformation at a Volcano***			
3				
4	*Given*			
5	Shear Modulus	30	(GPa)	
6	Magma chamber radius	0.5	(km)	
7	Depth	5	(km)	
8	Change in pressure	10	(MPa)	
9		Displacement		
10	Radial	Vertical	Horizontal	
11	Distance (km)	(m)	(m)	
12	0	0.00125	0	
13	0.5	0.001231482	0.000123148	
14	1	0.001178583	0.000235717	
15	1.5	0.001098425	0.000329527	
16	2	0.001000514	0.000400205	
17	2.5	0.000894427	0.000447214	
18	3	0.000788137	0.000472882	
19	3.5	0.000687275	0.000481093	
20	4	0.000595174	0.00047614	
21	4.5	0.000513325	0.000461992	
22	5	0.000441942	0.000441942	
23	5.5	0.000380471	0.000418518	
24	6	0.000327963	0.000393556	
25	6.5	0.000283323	0.00036832	
26	7	0.000245455	0.000343637	
27	7.5	0.000213346	0.000320019	
28	8	0.000186095	0.000297752	
29	8.5	0.000162924	0.000276971	
30	9	0.000143173	0.000257711	
31	9.5	0.000126287	0.000239945	
32	10	0.000111803	0.000223607	
33	11	8.85708E-05	0.000194856	
34	12	7.11197E-05	0.000170687	
35	13	5.78252E-05	0.000150346	
36	14	4.75589E-05	0.000133165	
37	15	3.95285E-05	0.000118585	
38	20	1.78335E-05	7.1334E-05	
39	25	9.42866E-06	4.71433E-05	
40	30	5.55402E-06	3.33241E-05	

Carrying out the Plan, Part 1: Surface Displacement Data

Given surface displacement data from Volcano X, estimate the depth and source strength of the magma chamber.

Searching for the relationship between pressure change and depth of the magma chamber requires the calculation of χ^2. The data used here were gathered by volcanologists prior to the 2010 eruption of Volcano X.

Copy and paste your spreadsheet from slide 10 onto a second worksheet. Insert two columns so that you can paste the data in the spreadsheet (to the right) into your spreadsheet.

	B	C	D
2	***What is the Depth of the Magma Chamber and***		
3	***the Change in Pressure for Volcano X?***		
4	*Given*		
5	Shear Modulus		(GPa)
6	Magma Chamber radius		(km)
7	Depth		(km)
8	Change in Pressure		(MPa)
9		Displacement	
10	Radial Distance	Vertical	Horizontal
11	(km)	(m)	(m)
12	0.56	0.01568	0.00118
13	0.65	0.01563	0.00136
14	0.9	0.01547	0.00187
15	1.2	0.01521	0.00245
16	1.56	0.01482	0.00310
17	2.5	0.01347	0.00452
18	2.75	0.01305	0.00482
19	3.15	0.01235	0.00522
20	3.76	0.01125	0.00568
21	4.23	0.01040	0.00590
22	5	0.00905	0.00607
23	5.35	0.00847	0.00608
24	5.98	0.00750	0.00602
25	6.26	0.00709	0.00596
26	6.87	0.00628	0.00579
27	7.14	0.00595	0.00570
28	7.79	0.00522	0.00546
29	8.12	0.00489	0.00532
30	9.35	0.00383	0.00480
31	9.5	0.00372	0.00474
32	10.2	0.00324	0.00444
33	11.2	0.00269	0.00404
34	12.5	0.00212	0.00356
35	14.7	0.00146	0.00288
36	15.2	0.00135	0.00275
37	15.7	0.00125	0.00263
38	19.78	0.00069	0.00184
39	25.4	0.00035	0.00120
40	32	0.00018	0.00079

To obtain the data in this spreadsheet, return to "normal view" in PowerPoint. Double click on the spreadsheet. A window will open where you can highlight the three columns of data and copy it. Then paste the data into your new spreadsheet (after inserting two columns).

Take a careful look at the *units* of displacement and radial distance!

Carrying out the Plan, Part 2: Calculating Pressure Change and Depth

Kiyoo Mogi used the theory of elasticity to explain the measured surface displacement before, during and after the eruptions of several volcanoes. You will use his equations to solve for the depth of the magma chamber and change in pressure.

Again, note the *units*! Pressure change is in MPa and the shear modulus is in GPa. Recall that Pa are equal to N m^{-2} or kg s^{-2} m^{-1}.

	B	C	D	E	F
2	***What is the Depth of the Magma Chamber and the Change***				
3	***in Pressure for Volcano X?***				
4	***Given***				
5	Shear Modulus	30	(GPa)		
6	Magma chamber radius	1	(km)		
7	Depth	5	(km)		
8	Change in pressure	20	(MPa)		
9		Displacement		Displacement	
10	Radial	Vertical	Horizontal	Vertical	Horizontal
11	Distance (km)	(m)	(m)	(m)	(m)
12	0.56	0.01568	0.00118	0.01963	0.00220
13	0.65	0.01563	0.00136	0.01950	0.00254
14	0.9	0.01547	0.00187	0.01907	0.00343
15	1.2	0.01521	0.00245	0.01839	0.00441
16	1.56	0.01482	0.00310	0.01740	0.00543
17	2.5	0.01347	0.00452	0.01431	0.00716
18	2.75	0.01305	0.00482	0.01345	0.00740
19	3.15	0.01235	0.00522	0.01211	0.00763
20	3.76	0.01125	0.00568	0.01021	0.00768
21	4.23	0.01040	0.00590	0.00890	0.00753
22	5	0.00905	0.00607	0.00707	0.00707
23	5.35	0.00847	0.00608	0.00637	0.00681
24	5.98	0.00750	0.00602	0.00528	0.00631
25	6.26	0.00709	0.00596	0.00486	0.00609
26	6.87	0.00628	0.00579	0.00408	0.00560
27	7.14	0.00595	0.00570	0.00377	0.00539
28	7.79	0.00522	0.00546	0.00315	0.00491
29	8.12	0.00489	0.00532	0.00288	0.00468
30	9.35	0.00383	0.00480	0.00210	0.00392
31	9.5	0.00372	0.00474	0.00202	0.00384
32	10.2	0.00324	0.00444	0.00171	0.00348
33	11.2	0.00269	0.00404	0.00135	0.00303
34	12.5	0.00212	0.00356	0.00102	0.00256
35	14.7	0.00146	0.00288	0.00067	0.00196
36	15.2	0.00135	0.00275	0.00061	0.00186
37	15.7	0.00125	0.00263	0.00056	0.00175
38	19.78	0.00069	0.00184	0.00029	0.00116
39	25.4	0.00035	0.00120	0.00014	0.00073
40	32	0.00018	0.00079	0.00007	0.00047

Change these values in your spreadsheet.

Be certain your formulas refer to the correct cells.

Cell with a number in it (variable). Change one of these numbers and other numbers will change.

Cell with data. These values should NOT be changed.

Cell with an equation

Carrying out the Plan, Part 3: Calculating Pearson's χ^2 test

Now calculate the χ^2 test to see how good your model fits the data. You will need to calculate χ^2 for each estimate of depth and pressure change and solve for the minimum, total (vertical and horizontal) χ^2 value. To do this you will want to hold one parameter fixed, while varying the other (green cells).

For an example of how to do this, see *Slide 17*.

	B	C	D	E	F	G	H
2	***What is the Depth of the Magma Chamber and the Change***						
3	***in Pressure for Volcano X?***						
4	***Given***						
5	Shear Modulus	30	(GPa)				
6	Magma chamber radius	1	(km)				
7	Depth	7.5	(km)		**Varied**		
8	Change in pressure	20	(MPa)		**Fixed**		
9		Displacement		Displacement			
10	Radial	Vertical	Horizontal	Vertical	Horizontal	Chi-Square:	Chi-Square:
11	Distance (km)	(m)	(m)	(m)	(m)	Vertical	Horizontal
12	0.56	0.01568	0.00118	0.00882	0.00066	0.00535	0.00041
13	0.65	0.01563	0.00136	0.00879	0.00076	0.00532	0.00047
14	0.9	0.01547	0.00187	0.00870	0.00104	0.00527	0.00065
15	1.2	0.01521	0.00245	0.00856	0.00137	0.00517	0.00085
16	1.56	0.01482	0.00310	0.00834	0.00174	0.00503	0.00107
17	2.5	0.01347	0.00452	0.00759	0.00253	0.00456	0.00157
18	2.75	0.01305	0.00482	0.00736	0.00270	0.00441	0.00167
19	3.15	0.01235	0.00522	0.00697	0.00293	0.00416	0.00180
20	3.76	0.01125	0.00568	0.00635	0.00318	0.00378	0.00196
21	4.23	0.01040	0.00590	0.00587	0.00331	0.00349	0.00202
22	5	0.00905	0.00607	0.00512	0.00341	0.00302	0.00207
23	5.35	0.00847	0.00608	0.00480	0.00342	0.00281	0.00207
24	5.98	0.00750	0.00602	0.00425	0.00339	0.00249	0.00205
25	6.26	0.00709	0.00596	0.00402	0.00336	0.00234	0.00202
26	6.87	0.00628	0.00579	0.00356	0.00326	0.00207	0.00195
27	7.14	0.00595	0.00570	0.00338	0.00322	0.00196	0.00192
28	7.79	0.00522	0.00546	0.00297	0.00308	0.00171	0.00184
29	8.12	0.00489	0.00532	0.00278	0.00301	0.00161	0.00178
30	9.35	0.00383	0.00480	0.00218	0.00271	0.00125	0.00160
31	9.5	0.00372	0.00474	0.00211	0.00268	0.00122	0.00159
32	10.2	0.00324	0.00444	0.00185	0.00251	0.00105	0.00148
33	11.2	0.00269	0.00404	0.00153	0.00229	0.00088	0.00134
34	12.5	0.00212	0.00356	0.00121	0.00202	0.00068	0.00118
35	14.7	0.00146	0.00288	0.00083	0.00164	0.00047	0.00095
36	15.2	0.00135	0.00275	0.00077	0.00156	0.00044	0.00091
37	15.7	0.00125	0.00263	0.00071	0.00149	0.00041	0.00087
38	19.78	0.00069	0.00184	0.00040	0.00104	0.00022	0.00061
39	25.4	0.00035	0.00120	0.00020	0.00068	0.00011	0.00039
40	32	0.00018	0.00079	0.00011	0.00045	0.00005	0.00026
				Sum Chi-square:		0.07131	0.03933
				Total chi-square:		0.11064	

What you have done

You have investigated the relationship between the pressure change in a magma chamber, the depth of the magma chamber and the vertical and horizontal surface displacements.

K. Mogi first utilized elasticity theory for the investigation of surface displacements caused by pressure and volume changes in magma chambers, buried in an elastic-half space, before, during and after eruptions. This theory and equations are still used today to investigate volcano deformation.

A lot is NOT considered in these simple elastic half-space models. For example, you have not considered more complex geometries like sills and dikes (horizontal and vertical tabular bodies, respectively) or cylinders, the physical properties of the magma and country rock, permanent (plastic) deformation of the crust and time-varying deformation. It is interesting that a reasonable correlation can be identified without considering these basic properties of the magma, magmatic system and volcano.

From a hazard perspective, models of this type are quite important. You have a general estimate of the location of the magma chamber and the magnitude of pressure prior to an eruption, based on observations that can be made on and off the volcano or using remotely sensed data. These can give us an indication of the potential explosivity of the eruption. Such information is valuable to people living on or near volcanoes!

There is much more to understanding volcano deformation. To get started, see:

Mogi, K., 1958, Relations between the eruptions of various volcanoes and the deformations of the ground surfaces around them, Bull. of the Earthquake Res. Inst., vol. 36, 99-134.

Delaney, P. and McTigue, 1994, Volume of magma accumulation or withdrawal estimated from surface uplift or subsidence, with application to the 1960 collapse of Kilauea volcano, Bull. Volc., 56, 417-424.

Johnson, D., Sigmundsson, F., and Delaney, P., 2000, Comment on “Volume of magma accumulation or withdrawal estimated from surface uplift or subsidence, with application to the 1960 collapse of Kilauea volcano,” Bull. Volc., 61, 491-493.

End-of-Module Assignments

1. Turn in a spreadsheet showing your calculation of the relationship between *pressure change* and vertical and horizontal surface displacements Slide 10.

2. Turn in a spreadsheet showing your calculation of the relationship between *volume* change and vertical and horizontal surface displacements. Use a volume change of 1 km^3.

3. Consider the equation for vertical displacement, U_z, and the range of parameter values given on slide Slide 7 for change in pressure, depth, and radius of the magma chamber. Is vertical displacement linearly, or non-linearly dependent on each of these three parameters? Plot the change in *maximum* vertical displacement as a function of depth (3-10 km) for constant pressure (30 MPa) and constant radius (1 km). Plot the change in maximum vertical displacement as a function of magma chamber radius (500-1000 m) for constant magma chamber depth (1 km) and pressure (30 MPa). Given your results and your understanding of magma ascent, what is most likely causes a change in vertical displacement of the surface before a volcanic eruption?

4. Calculate and show plots for the best-fit parameters (i.e., minimum $\boldsymbol{\chi^2}$ for change in pressure and depth) that fit the displacement data for Volcano X Slide 13 & 17. Plot χ^2 versus change in pressure and versus depth. Are there more than one possible combinations of pressure change and depth for the surface displacements measured at Volcano X? (Assume radius = 1 km and shear strength = 30 GPa; these are held constant!) Use the range of depths and pressures from Slide 7.

5. Calculate and show plots for the best-fit parameters (i.e., minimum $\boldsymbol{\chi^2}$ for change in volume and depth) that fit the displacement data for Volcano X. (Assume radius = 1 km and is held constant!). Use a range of ΔV from 0.5 to 5 km^3.

6. Is there a difference between the estimated depths? What may cause this difference?

How Did We Obtain Volume Change from Pressure Change?

$$\Delta V = \frac{\pi a^3 \Delta P}{G}$$

(i.e., volume change, ΔV, is proportional to pressure change, ΔP, and inversely proportional to shear modulus (G).

$$a^3 \Delta P = \frac{G \Delta V}{\pi}$$

Rearrange and substitute into equations for U_z and U_r *for pressure change.*

$$U_z = \frac{3a^3 \Delta P d}{4G(d^2 + r^2)^{1.5}} = \frac{3G \Delta V d}{4G\pi(d^2 + r^2)^{1.5}} = \frac{3 \Delta V d}{4\pi(d^2 + r^2)^{1.5}}$$

$$U_r = \frac{3a^3 \Delta P r}{4G(d^2 + r^2)^{1.5}} = \frac{3G \Delta V r}{4G\pi(d^2 + r^2)^{1.5}} = \frac{3 \Delta V r}{4\pi(d^2 + r^2)^{1.5}}$$

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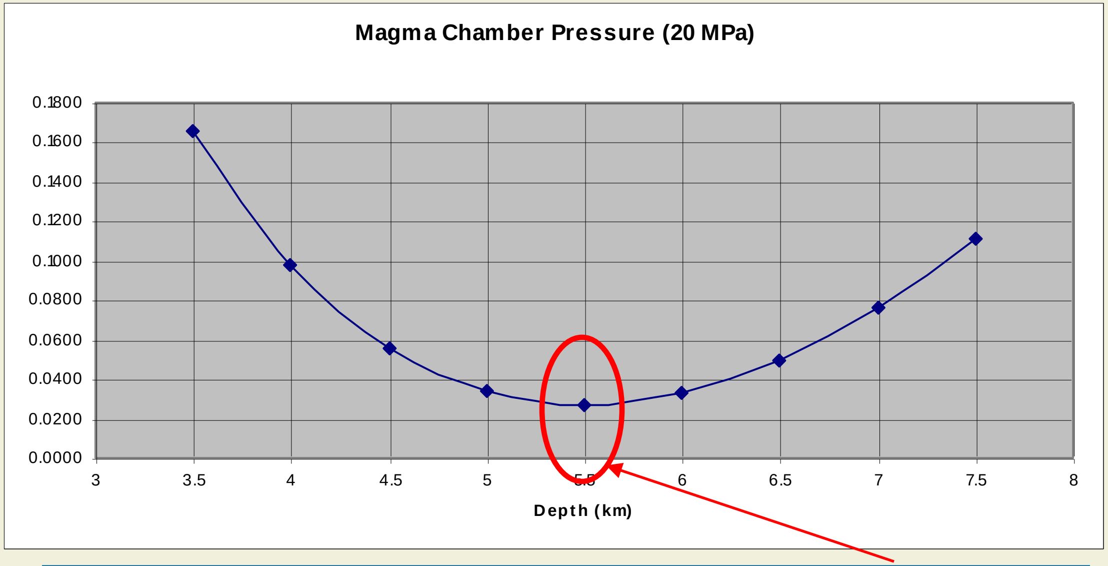
Calculating the Minimum Pearson's χ^2 Test

Using the equation on page 9 and the spreadsheet from page 13, calculate the minimum, total Pearson's χ^2 for the parameters of pressure change and magma chamber depth. To the right is an example. In the spreadsheet on page 13, the change in magma chamber pressure is held constant at 20 MPa and the magma chamber depth is varied. For each depth, calculate the total χ^2. Now change the pressure (e.g. to 25 MPa) and recalculate the best-fit models by varying the depth.

Pressure (20 MPa)	
Depth (km)	total chi-square
3.5	0.1660
4	0.0975
4.5	0.0557
5	0.0335
5.5	0.0268
6	0.0326
6.5	0.0493
7	0.0756
7.5	0.1106



The minimum point on this graph (5.5 km) best describes the depth of the magma chamber at 20 MPa pressure.

Do you want to learn more about χ^2 :
http://www2.chass.ncsu.edu/garson/pa765/chisq.htm

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References

Figures on page 3 are from:
http://hvo.wr.usgs.gov/howwork/subsidence/inflate_deflate.html

Photo of Mount St. Helens bulge page 4 from:
http://volcanoes.usgs.gov/About/What/Monitor/Deformation/MSHDfrm.html

Data figure page 4 from: http://hvo.wr.usgs.gov/howwork/subsidence/main.html

Amelung, F., Jonsson, S., Zebker, H., Segall, P., 2000. Widespread uplift and “trapdoor” faulting on Galapagos volcanoes observed with radar interferometry, Nature, 407, 993-996.

Additional References:

Mogi, K., 1958. Relations between the eruptions of various volcanoes and the deformation of the ground surface around them, *Bull. Earthquake Res. Inst*., v. 36, 99-134.