Bubbles in Magmas

How do bubbles grow by decompression in silicate magmas? How fast do they rise?

Application of the ideal gas law to bubble growth and bubble rise

Core Quantitative Issue
Forces

Supporting Quantitative Issues
Ideal Gas Law
Units
Slides 3-6 give some background on bubbles in magmas and discuss how bubbles are intimately related to magma flow and dynamics.

Slide 7 states the first problem. How do isolated bubbles grow by decompression and rise in magma?

Slides 8-11 analyze this problem. The solution involves the ideal gas law and resolving the forces acting on a rising bubble.

Slide 12 illustrates a spreadsheet that calculates a solution to this problem.

Slide 13 summarizes the module and points to open issues in modeling bubble behavior in silicate magmas.

Slide 14 gives the end-of-module assignments.

Slides 15 and 16 provide endnotes and additional references.
Background

Why do bubbles form in silicate magmas?

Volatile compounds, mainly water and carbon dioxide, are completely dissolved in magmas under high pressure conditions deep within the earth. In other words, magmas under high pressure are under-saturated with these volatile compounds. As the magma rises, the solubility of volatiles decreases, the magma becomes saturated or perhaps super-saturated with respect to volatiles, and bubbles begin to grow. Exactly where on the journey to the surface this happens depends on the amount of volatiles dissolved in the magma, the solubility of each specific volatile in the magma, and additional factors, such as the presence of crystals that may help bubbles form.

This bubble in basaltic magma was photographed just as it burst, having risen to the surface of the Mauna Ulu lava lake during the 1969 eruption. The bubble diameter at the surface is approximately five meters. (Photo courtesy of the USGS).

Background

Why are bubbles important in volcanology?

Nucleation of bubbles and bubble growth in some magmas accelerates ascent and causes explosive eruptions.

Consider a volcanic conduit with cross-sectional area:

\[ a = \pi r^2 \]

and mass flow \( Q \). As pressure decreases higher in the conduit, bubbles grow. The bubbles lower the density of the ascending magma. Conservation of mass dictates:

\[ Q = \rho au \]

where \( \rho \) is the density of the magma (including bubbles) and \( u \) is the ascent velocity of the mixture (assuming the bubbles rise at the same speed as the rest of the magma). So as density drops, ascent velocity must increase, in order for \( Q \) to remain constant.

An explosive eruption at St. Augustine volcano, Alaska, during 1986. In this eruption, sustained mass flow of magma from a magma reservoir at depth, coupled with bubble nucleation and expansion, caused a sustained explosive eruption (photo courtesy of the USGS).

Be sure you understand the concept of conservation of mass. Check the units of \( Q \). The mass flow is the same at the top and bottom of the conduit, but density and velocity are different.
Background

Why are bubbles important in volcanology?

At many volcanoes, magma degases passively – in such cases the rate of gas escape from the magma exceeds the rate of magma rise, at least at the surface.

Passive degassing at Villarrica volcano, Chile

Photo by P.C. LaFemina

Passive release of volcanic gases into the atmosphere is a dramatic end-product of bubble nucleation and rise in magmas. Such degassing is the most common form of volcanic activity and is the primary source of Earth’s atmosphere (Masaya volcano, Nicaragua – one of Earth’s most persistently degassing volcanoes).

In this module, you will concentrate on passive degassing.
In passive degassing (i.e., gas flow from the volcano with very little or no eruption of magma), bubbles of gas must move toward the surface and escape. Consider two methods of accomplishing this without erupting large amounts of magma.

**Method 1:** Individual bubbles rise through the magma due to buoyancy force. As they rise, they grow primarily by decompression. When these bubbles reach the surface, they break, releasing gas and perhaps throwing volcanic bombs as the wall of the bubble bursts into the air. Such a mechanism requires a low viscosity magma and will not sustain high gas flux.

**Method 2:** Bubbles ascend the conduit together with magma \( (Q) \). As bubbles rise, they grow by decompression. Bubble density (number of bubbles per cubic meter) is high enough for bubbles to touch and connect as they ascend and grow. This connection creates permeability and allows gas to escape. As the bubbles degas, they lose pressure and collapse, increasing the density of the magma and causing it to sink, perhaps at the margins of the conduit \( (q) \). Viscous magmas may degas in this manner, as long as mass flow is sustained from depth. Such a mechanism can sustain very high gas flux.
Given a bubble of water vapor (10 g) at some depth in a lava lake, what is the change in radius of the bubble and rise rate of the bubble as it ascends through the lava lake toward the surface? *(Magma temperature is 1300 K, viscosity is 1000 Pa s, density is 2500 kg m⁻³)*

In this example, you will specify properties of the gas bubble (e.g., moles of water vapor in the bubble) in order to quantify the bubble ascent. You will consider only individual bubbles in isolation, rather than groups of bubbles for this problem.

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<td>7</td>
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<td>Temperature</td>
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Start a spreadsheet by entering the physical constants and given information for the problem. Calculate the number of moles of water vapor.
You will need to
• Convert the mass of water vapor into moles.
• Use the ideal gas law to calculate the volume and radius of the bubble.
• Calculate the buoyancy force, viscous drag, and terminal velocity of the bubble.

Notes:
(1) In this example you will only consider the affects of decompression. Leave the problems of bubble nucleation and diffusion of gas into bubbles from the magma for another time!
(2) Consider an isolated bubble (method 1 for passive degassing on slide 6).
(3) In order to fully solve the problem, you will need to resolve all the forces acting on an individual bubble.

Given a mass of water vapor in a bubble in magma, what is the bubble radius and ascent rate?

Give answer in units of meters for radius and meters per second for rise rate of the bubble.
Using the ideal gas law to find bubble volume

Water vapor, and any other gas, is compressible. That is, at higher pressures a given number of moles of water vapor takes up less volume than at lower pressures. The volume occupied by a gas at some pressure and temperature is reasonably approximated by the ideal gas law.

\[ PV = nRT \]

where \( P \) is pressure, \( V \) is volume, \( n \) is moles of the gaseous substance, \( R \) is the gas constant and \( T \) is temperature.

Before you go further with the spreadsheet, try this:

What is the volume of a water vapor bubble (\( n=1 \) mol) at 10 m depth in a lava lake (\( T= 1473 \) K, \( \rho = 2500 \) kg m\(^{-3}\))?

1. Visualize the problem:

What is the bubble radius with fixed \( T, n \)?

2. Consider the variables: \( P, n, R, T \)
   
   Given \( n, T \)
   
   \( R \) is constant
   
   \( P=\rho gh \)

3. Solve for \( P \)
4. Solve for \( V \)
5. Evaluate your answer. Does it make sense? What happens to the bubble volume at shallower depth (e.g., 5 m) or greater depth (e.g., 50 m)?

More review on the ideal gas law:

http://hyperphysics.phy-astr.gsu.edu/hbase/kinetic/idegas.html
What is the terminal velocity of an ascending bubble?

Three forces are acting on the ascending bubble.

Buoyancy force imparted by the density contrast between the bubble and the magma:

\[ F_{buoyancy} = m_{bubble} g \left( \frac{\rho_{magma}}{\rho_{bubble}} \right) \]

Gravitational force due to the weight of the bubble:

\[ F_{gravity} = m_{bubble} g \]

Viscous force due to the drag imposed by magma flowing around the rising bubble:

\[ F_{viscous} = 6\pi r \eta u \]

where \( m \) is mass, \( g \) is gravity, \( \rho \) is density, \( r \) is the bubble radius, \( \eta \) is viscosity of the magma, and \( u \) is the ascent velocity of the bubble.
What is the terminal velocity of an ascending bubble?

The net force acting on the bubble is:

\[ F_{\text{net}} = F_{\text{buoyancy}} - F_{\text{gravity}} - F_{\text{viscous}} \]

When the net force is positive, the bubble is accelerating upward. When the net force is zero, the bubble is not accelerating, but has reached its terminal velocity.

Recast the equation for net force so you can solve for the terminal velocity of a bubble rising in magma. Use the equations on Slide 10.
In a spreadsheet, the calculation looks like:

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Using information from the previous slides, decide what to enter in each cell containing a formula.
What you have done

You have investigated decompression and rise of bubbles in magmas.

Bubbles play a crucial role in volcanology. The formation and expansion of bubbles accelerates flows, and carries gas to the Earth’s surface in bubbles, ultimately creating the Earth’s atmosphere. Here we have dealt in detail with the decompression of bubbles, but the topic is even more complex. The physics of bubble nucleation (where, when, and why bubbles form) is another extremely important facet of the story. Furthermore, bubbles grow by diffusion of gas from the melt into the bubble. This is another important factor governing the nature of bubble growth. In this module, we have assumed that equilibrium conditions prevail (for example that the bubble pressure will equilibrate with local hydrostatic pressure). In fact, this is not necessarily the case. It is fair to say that the study of bubbles is a rich and active field of research in physical volcanology.

You have discovered that it is possible to study bubble decompression by considering the thermodynamics (ideal gas law) and physics (net force acting on bubbles) of bubble ascent using simplified assumptions. Often such models are used in the real world to understand basic processes and as a starting point for more physically realistic (and often much more challenging!) models. Natural processes are complex – learn to simplify!

Some useful starting points for learning more about bubbles in magmas:


* Cashman, K.V. and Mangan, M.T., 1994, Physical aspects of magmatic degassing II. Constraints on vesiculation processes from textural studies of eruptive products. Reviews in Mineralogy, 30: 447-478. [a starting point for understanding bubbles in rocks]
1. Make sure you turn in your spreadsheet showing the worked example.

2. Carbon dioxide and sulfur dioxide are common volatiles in magmas. Recast your spreadsheet to calculate the radius and terminal velocity of carbon dioxide and sulfur dioxide bubbles as a function of depth in the lava lake. Use 10 g bubbles. Make sure you use the correct molecular mass for CO₂ and SO₂. See, for example: http://en.wikipedia.org/wiki/Carbon_dioxide. Describe how and why this change in composition (H₂O, CO₂, SO₂) changes the bubble radius and ascent velocity. Of course, bubbles in lava lakes generally contain mixtures of these compounds.

3. The same forces that describe bubble rise through magma describe the fall of a pyroclast through the air (Slides 10 and 11). There are complexities: the density of the atmosphere changes dramatically as a function of height, and the equation for viscous drag is generally for turbulent flow, rather than laminar flow. (a) Use the ideal gas law to calculate the change in density of the atmosphere as a function of height from sea-level (0 m) to 30 km (approximately the height of a very large Plinian eruption column). Assume atmospheric pressure changes with height as:

\[ P(h) = P_{\text{sea-level}} \exp[-gh/RT] \]

where \( P_{\text{sea-level}} \) is atmospheric pressure at sea-level, \( g \) is gravity, \( R=285 \text{ J kg}^{-1} \text{ K}^{-1} \) is the gas constant, and \( T=273 \text{ K} \) – assume temperature does not change with height. Draw a graph of your results and turn in your spreadsheet. (b) Assume the drag force acting on a falling pyroclast is:

\[ F_v = \frac{1}{2} \pi r^2 u^2 C_d \rho_a \]

where \( r \) is the particle radius, \( u \) is the terminal velocity, \( C_d=0.44 \) is a drag coefficient, and \( \rho_a \) is the density of the atmosphere. Write out the equation for terminal velocity and calculate the terminal velocity for a 1 cm diameter pyroclast (density of 600 kg m⁻³) as a function of height from 30 km to sea-level. Draw a graph and discuss your results! (Hint: you will find you can neglect the buoyancy force!)
Buoyancy force arises because pressure increases with depth and because pressure acts on all sides of a body, such as a bubble rising through magma or a pumice rising through water.

The buoyancy force is equal to the weight of the fluid displaced by the object:

\[ F_{\text{buoyancy}} = \rho_{\text{magma}} V_{\text{object}} g \]

where \( \rho \) is density, \( V \) is volume, and \( g \) is gravity. Note that this means that the object experiences buoyancy force regardless of the object’s density (or whether the object rises or sinks in the fluid).

Given the above equation, prove to yourself that another way to express the buoyancy force is:

\[ F_{\text{buoyancy}} = \frac{\rho_{\text{magma}}}{\rho_{\text{object}}} m_{\text{object}} \]

where \( m \) is mass.

More about buoyancy force: [http://hyperphysics.phy-astr.gsu.edu/Hbase/pbuoy.html](http://hyperphysics.phy-astr.gsu.edu/Hbase/pbuoy.html)
When flow passed the object is laminar (not turbulent), then the drag force is proportional to velocity:

$$F_{\text{drag}} = -bu$$

where $b$ is a constant and $u$ is velocity (the negative sign means the force acts in the direction opposite velocity).

Stokes discovered that for laminar flow around a sphere:

$$b = 6\pi \eta a$$

where $\eta$ is viscosity and $a$ is the radius of the sphere.