Part III: InSAR Deformation Modeling

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• Original SAR data are copyrighted ESA, CSA, JAXA, or DLR
Volcano Deformation

1. Many volcanic eruptions are preceded by pronounced ground deformation in response to increasing pressure from magma chambers or to the upward intrusion of magma.

2. Surface deformation patterns can provide important insights into the structure, plumbing, and state of restless volcanoes.

3. Surface deformation might be the first sign of increasing levels of volcanic activity, preceding swarms of earthquakes or other precursors that signal impending intrusions or eruptions.

4. Surface deformation provides a critical element on understanding how a volcano works.
Deformation Source

**deformation:**
what we see (InSAR)

**magma dynamics:**
what we want to know

Magma intrusion
Deformation modeling

Spherical point source (Mogi source)

\[ u_i (x_1 - x'_1, x_2 - x'_2, 0) = \frac{(1 - v)}{\pi} \frac{x_i - x'_i}{R^3} \Delta V \]

where \( x'_1, x'_2, \) and \( x'_3 \) are horizontal locations and depth of the center of the sphere, \( R \) is the distance between the sphere and the location of observation \((x_1, x_2, \) and 0), and \( v \) is the Poisson’s ratio of host rock.

Best-fit source parameters:
• The model source is located at a depth of 6.5 ± 0.2 km.
• The calculated volume change of magma reservoir is 0.043 ± 0.002 km³.
Estimate source characteristics from InSAR deformation data

**Forward model**
- Design matrix: $G$
- Source parameters: $s$
- Displacement (vector): $d$

$$G \circ s = d$$

**Inverse model**

$$s = G^{inv}d$$
Linear Inversion

If the covariance matrix for errors in the observation \((b)\) is \(\sum_b\), then the weighted least-squares (maximum likelihood) solution for \(x\) is

\[
\hat{x} = \left[ G^T \sum_b^{-1} G \right]^{-1} \left[ G^T \sum_b^{-1} b \right]
\]

The covariance matrix for the estimated vector components is

\[
\sum_x = \left[ G^T \sum_b^{-1} G \right]^{-1}
\]

In the case where we assume that observation errors are independent and have equal standard deviations, \(\sigma\), we get

\[
\sum_x = \sigma^2 \left[ G^T G \right]^{-1}
\]

The square root of the diagonal terms give the standard errors in parameter estimates.
Predicts deformation \( \mathbf{u} \) caused by magma intrusion (relates magma intrusion to deformation)

\[ \mu \nabla^2 \boldsymbol{u}_i + \frac{\mu}{(1-2\nu)} \left[ \frac{\partial^2 \boldsymbol{u}_k}{\partial x_i \partial y_k} \right] = -F_i \]

**elasto-static behavior**

\[ \mathbf{u} = f(\text{model parameters}) \]
Forward model: point source

A component of deformation vector \( (u_i) \) and the displacement at the free surface \( (x_3=0) \) takes the form

\[
u_i(x_1 - x_i', x_2 - x_2', -x_3) = C \frac{x_i - x_i'}{|R^3|}
\]

\( x_i' \) is a source location, \( C \) is a combination of material properties and source strength, and \( R \) is the distance from the source to the surface location.

\[
C = \Delta P (1 - \nu) \frac{r_s^3}{G} = \Delta V \frac{(1 - \nu)}{\pi}
\]

\( \Delta p \) - change in pressure of magma chamber
\( \Delta V \) - change in volume of magma chamber
\( \nu \) - Poisson’s ratio
\( r_s \) - radius of the sphere
\( G \) - shear modulus of country rock
Forward model: point source

\[ \alpha \ll d \]

D. Dzurisin, 2007

Courtesy of M. Lisowski
Forward model: spherical source

- Vertical
  - $\alpha/d = 0.6$
  - $\alpha/d = 0.4$
  - $\alpha/d = 0$ (point source)

- Horizontal

D. Dzurisin, 2007
Courtesy of M. Lisowski
Forward model: closed pipe

Isotropic elastic half-space \((G, \nu)\)

Closed pipe, prolate spheroid \((a = b < c)\)

\(c_1\)

\(c_2\)

Vertical

Horizontal

\(C_2 \rightarrow \text{infinity}\)

D. Dzurisin, 2007

Courtesy of M. Lisowski
Forward model: closed pipe

isotropic elastic half-space \((G, \nu)\)

closed pipe, prolate spheroid \((a = b \ll c)\)

\[ \begin{align*}
    C_2 &= 10C_1 \\
    C_2 &= 20C_1 \
\end{align*} \]

Vertical: \(C_2 \to \infty\)

Vertical: \(C_2 = 10C_1\)

Vertical: \(C_2 = 20C_1\)

Horizontal

D. Dzurisin, 2007

Courtesy of M. Lisowski
Forward model: pipe vs Mogi

D. Dzurisin, 2007
Forward model: open pipe

Constant pressure change in the lower section of conduit

Filling the top portion of the conduit from c1 to surface

Combined effect of filling a conduit from c1 to surface

\[ b = \frac{\Delta P}{G} \alpha \]

Courtesy of M. Lisowski

D. Dzurisin, 2007
Forward model: sill

isotropic elastic half-space $(G, \nu)$

Vertical
Horizontal

Displacement

spherical pressure source

Courtesy of M. Lisowski
Forward model: dike

D. Dzurisin, 2007
Courtesy of M. Lisowski
Forward model

A complex example: viscoelastic shell surrounding magma chamber

![Diagram showing a viscous and elastic shell surrounding a magma chamber.](image)

**Viscoelastic Chamber** $R_2/R_1 = 1.2$

- **Vertical**
  - $t_{A_R} = 0$
  - $t_{A_R} = 1$
  - $t_{A_R} = 3$
  - $t_{A_R} = 10$

- **Horizontal**
  - $0$ to $3$

Courtesy of P. Segall
Deformation Source Models

Simple Source Models in Elastic Half-Space
- Spherical Point Source
- Prolate Ellipsoid
- Sill or Dike for volcanoes
- Penny-shaped Sill
- Pipe
- Dislocation for earthquakes

Complicating Effects
- Non-uniform Elastic Structure
- Topography
- Viscoelasticity
- Poroelasticity
- Thermoelasticity
- Complex Geometry
- Influence of hydrothermal fluid

\[ u = f(\text{model parameters, material properties, ..., }) \]
Ultimate Goal of Deformation Modeling

Minimize

\[ \sum [u_i(x, y) \cdot los_i(x, y) - obs_i(x, y)] \]

- \( u_i \) is a theoretical calculation of ground surface deformation vector (i=1, 2, 3)
- \( los_i \) is the InSAR line-of-sight vector
- \( obs_i \) is the observed deformation (InSAR image)
- \((x, y)\) is the image coordinate

Non-linear inversion!!!!
Find best-fit model parameters

1. loop through model parameters
   • calculate the residual (observed – modeled) for each set of model parameters

2. find the set of model parameters that renders the smallest residual
   => best-fit model parameters
A simple matlab code for deformation modeling

% Mogi_modeling.m
% define upper bounds of source parameters
ub = [ 21.6 23.2 7.0 -0.03 50 25 25];
    X       Y       Z       ΔV     static term (or baseline_error_terms)
% define lower bounds of source parameters
lb = [ 19.6 11.2 2.0 -0.08 -50 -25 -25];

% READ “InSAR image and InSAR geometry parameters“

SIMULATIONS = 10;
A simple matlab code for deformation modeling

% **Mogi_modeling.m** (cont’d)

for i=1:SIMULATIONS
    % generate random numbers between 0 and 1.0;
    rand_vec=rand(1, source_parameter_length);
    diff_vec=ub - lb;
    p_start=lb + diff_vec*rand_vec;
    [p_new, RESNORM, residual, EXITFLAG]=…
    lsqnonlin('mogi_func', p_start, lb, ub, opts_in);
end

% **LSQNONLIN** solves non-linear least squares problems.

% **LSQNONLIN** attempts to solve problems of the form:

\[
\min \sum\{|\text{FUN}(X)|^2\}
\]

% where X and the values returned by FUN (new X) can be vectors or matrices.
A simple matlab code for deformation modeling

```matlab
% mogi_func.m
function [residual] = mogi_func(X);
% This function will return a matrix of the residual (difference between the data
% and calculated range change).
%
% USEAGE: [residual] = mogi_func(X);
% INPUT: X is a vector of Mogi source parameters
% OUTPUT: residual == a vector of observed data values minus modeled.

global eing_vec ning_vec obs_phase plook

calc_phase=rngchn_mogi(X(2),X(1),X(3),X(4), ning_vec,eing_vec,plook);

residual= obs_phase – calc_phase +X(5);
```

forward model
% rngchn_mogi.m (forward model)

- function [rng_change]=rngchn_mogi(n1,e1,depth,del_v,ning,eing,plook);
- % USEAGE: [rng_change]=rngchn_mogi(n1,e1,depth,del_v,ning,eing,plook);
- % INPUT:
- % n1 = local north coord of center of Mogi source (km)
- % e1 = local east coord of center of Mogi source (km)
- % depth = depth of Mogi source (km).
- % del_v = Volume change of Mogi source (km^3)
- % ning = north coord's of points to calculate range change
- % eing = east coord's of points to calculate range change
- % OUTPUT: rng_change = range change at coordinates given in ning and eing.

\[
u_i(x_1 - x'_1, x_2 - x'_2, -x_3) = \Delta V \frac{(1 - \nu)}{\pi} \frac{x_i - x'_i}{|R^3|}
\]
Multiple Sources

- Superimposition of individual deformation sources
- Smoothing (spatial + temporal)
The total displacement on a given patch...

...is related to that of patches adjacent to it, by a finite-difference Laplacian approximation:

\[(a_2 - a_5) - (a_5 - a_8) + (a_4 - a_5) - (a_5 - a_6) = 0\]

\[a_2 + a_4 - 4a_5 + a_6 + a_8 = 0\]

(schematic)
Source parameter error estimates

• One approach of estimating parameter errors is Monte Carlo simulation of correlated noise (Wright, Lu & Wicks, 2003).

• Multiple sets of correlated noise are simulated that have the same covariance function as observed in the data.

• A number of such data sets are added to the observation (e.g., InSAR phase changes).

• Parameter errors are determined from the distribution of the best-fit solutions to each of these noisy data sets.
Volcano structure

Basic concepts

standard model

required assumptions:
- homogeneous material properties
- isotropic material properties
- Poisson-solid
- half-space
Finite element models

Simulate volcano structures

caldera

magma

crust

mantle

\[ \mu \nabla^2 u_i + \frac{\mu}{(1-2\nu)} \left[ \frac{\partial^2 u_k}{\partial x_i \partial y_k} \right] = -F_i \]

elasto-static behavior

Courtesy of T. Masterlark
Example 1
Dynamic deformation of Seguam volcano


Multi-temporal InSAR Images

Masterlark & Lu, 2004
Deformation Modeling

point expansion source array

InSAR image having complex pattern

\[ d_j = s_i \left( \frac{-z_i}{R_{ij}^3} \right) \]

- \( d_j \): displacement
- \( s_i \): source strength
- \( z_i \): depth
- \( R_{ij} \): distance between source and point
- \( G_{ij} \): deformation parameters

north  up  east
Three clusters dominate, each having a distinctive time-dependent behavior

Masterlark & Lu, JGR, 2004
Dominant Source Clusters

Three clusters dominate, each having a distinctive time-dependent behavior.

potential point sources…

Masterlark & Lu, JGR, 2004
Transient deformation

- Cluster 1
- Cluster 2
- Cluster 3

Graphs showing source strength, eruption dates, and year.
Example 2
Afar – triple junction.
Smith and Cann, JGR, 2000
Quaternary strain localised to ~60 km long zones of fissures, aligned eruptive centers and faults - “magmatic segments “

Courtesy of T. Wright
14/9/2005 to 11/05/2005

163 earthquakes (mb <6) detected by NEIC.

Relocated by Anna Stork

Courtesy of T. Wright
3D displacements measured from radar data

Deflating Magma Chambers

Collapsed Zone along Rift

Collapsed Zone along Rift

Cross section
• 2.2 km$^3$ magma intruded along dyke (Mt St Helens 1980 1.2 km$^3$)
• 0.5 km$^3$ sourced from Dabbahu and Gabho volcanoes at North.
• Earthquakes can be responsible for < 10 % of moment release.

Wright et al., Nature, 2005
Example 3
Oct. 23 and Nov 3, 2002 Denali Earthquakes
2002 Denali Fault Earthquakes
InSAR images: observed and modeled

Oct. 23, 2002 Earthquake

Lu, Wright, Wicks, EOS, 2003
Slip Distribution of Oct 23, 2002 Earthquake

- Wright, Lu, Wicks, GRL, 2003
• One approach of estimating parameter errors is Monte Carlo simulation of correlated noise (Wright, Lu & Wicks, 2003).

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• Parameter errors are determined from the distribution of the best-fit solutions to each of these noisy data sets.
Model Parameter Error Bounds

Note location errors << 1 km

Wright, Lu and Wicks, GRL 30 (18), 2003
Example 4
A large and tragic collapse occurred in the Crandall Canyon coal mine on 6 Aug. 2007, causing the loss of 6 miners.

This collapse was accompanied by a local magnitude (M_L) 3.9 seismic event having a location and origin time coincident with the collapse (within current uncertainty limits).
Ground Surface Deformation From InSAR

08/06/2007 mainshock

center area of subsidence
LOS deformation of 20-25 cm
- the epicenter from the standard relocation program.
- the epicenter from a localized velocity structure.
- the epicenter from the master-event method.
- the epicenter from the double-difference relocation method.
- the damaged area by the MSHA
The sharp break in phase gradient on the south edge of the deformation signal is an important observation that is diagnostic of more than just a simple collapse model for the deformation source.

InSAR data are parsed using a quad-tree algorithm.

Deformation is modeled with distributed dislocation (Okada) sources.

An adequate model is defined as one for which the variance of the residual (observed data minus calculated) is reduced to the same variance as the noise in the non-deforming area of the interferogram.
An adequate fit is only found where the depth of flat lying sources is less than ~100 m. The mine depth is known to be around 500 m. Therefore, a simple collapse model with spatially varying collapses cannot explain the deformation field seen in the interferogram.
Modeled Deformation
Collapse sources + 40°-dipping fault

- constraining the depth of flat lying collapse sources to be 500 m
- adding a shallow uniform slip normal fault that dips to the north.
Modeled Deformation
Collapse sources + 65°-dipping fault

- constraining the depth of a flat lying collapse source to be 500 m
- adding a shallow uniform slip normal fault that dips to the north.
Modeled Deformation
Collapse sources + a normal fault

• We cannot well constrain the dip of the normal fault component of the model.

• At the 95% confidence level, a dip between $10^\circ$ and $85^\circ$ provides adequate fit.

• The top of the fault is shallow, shallower than 70 m and deeper than 20 m.

• The ratio between the normal fault and the collapse component decreases from about 2.5 at $20^\circ$ dip to 0.3 at a dip of $85^\circ$; however, a model with a dip of $85^\circ$ for a normal fault is too steep to intersect the modeled collapse area.

• The estimated geodetic moment (Mw4.5) is larger than seismic moment (Mw4.1).
Our favorite model

Lu & Wicks, 2010
Volcano Deformation

This book describes the techniques used by volcanologists to successfully predict several recent volcanic eruptions by combining information from various scientific disciplines, including geodetic techniques. Many recent developments in the use of state-of-the-art and emerging techniques, including Global Positioning System and Synthetic Aperture Radar Interferometry, mean that most books on volcanology are out of date, and this book includes chapters devoted entirely to these two techniques.